

Divergent Summability and Resurgence: Unraveling the Convergence of Divergent Series

The world of mathematics is replete with fascinating paradoxes and contradictions, and the concept of divergent summability stands as a prime example. It challenges our intuitive understanding of convergence and opens up new avenues for exploring the behavior of seemingly divergent series.

Divergent Summability: A Journey Beyond Convergence

In the realm of mathematics, a series is considered convergent if its partial sums approach a finite limit as the number of terms tends to infinity. However, some series, despite appearing to diverge, can possess a well-defined "sum" when approached using certain summation methods. This is the essence of divergent summability.



Divergent Series, Summability and Resurgence I: Monodromy and Resurgence (Lecture Notes in Mathematics Book 2153) by Claude Mitschi

★★★★☆ 4.8 out of 5

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The most well-known method of divergent summability is Borel summation. Introduced by Émile Borel in 1899, it assigns a unique value to divergent series by applying an exponential weighting to its terms.

Resurgence Theory: Unveiling Hidden Structures

Resurgence theory is closely intertwined with divergent summability. It delves into the analytic properties of divergent series and reveals hidden patterns that emerge under the lens of Borel summation.

When a divergent series is Borel resummed, it gives rise to a function called the Borel transform. Resurgence theory explores the singularities and asymptotic behavior of the Borel transform, uncovering connections between the series and complex analytic structures.

Analytic Continuation: Bridging the Gap

One of the remarkable aspects of resurgence theory is its ability to analytically continue divergent series into complex domains. This means that a divergent series, which initially appears meaningless, can be given a rigorous analytic interpretation in a wider context.

Analytic continuation allows us to study complex phenomena that cannot be captured by real-valued analysis alone. It provides insights into the behavior of divergent series beyond their apparent divergence.

Applications in Asymptotic Analysis

Divergent summability and resurgence theory find wide applications in asymptotic analysis, particularly in the study of integrals and differential equations. They provide powerful tools for understanding the behavior of

functions in the limit and for extracting meaningful information from seemingly divergent expressions.

For example, divergent summability can be used to calculate asymptotic expansions of integrals and to investigate the asymptotic behavior of solutions to differential equations. It enables us to obtain precise asymptotic results even when the underlying series appear to diverge.

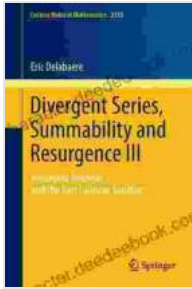
Examples and Case Studies

To illustrate the power of divergent summability and resurgence theory, consider the following examples:

- The divergent series $1 - 1 + 1 - 1 + \dots$ can be Borel summed to give the value $1/2$. This result has deep implications in areas such as statistical mechanics and probability theory.
- Resurgence theory has been applied to analyze the asymptotic behavior of the error function $\text{erf}(z)$. By understanding the singularities and asymptotic structure of the Borel transform of $\text{erf}(z)$, researchers have gained valuable insights into its complex behavior.

Divergent summability and resurgence theory open up new avenues for understanding the behavior of divergent series. They provide powerful tools for exploring complex phenomena and for obtaining precise asymptotic results. These concepts have far-reaching implications in mathematics, physics, and other scientific disciplines, offering a glimpse into the hidden structures of our universe.

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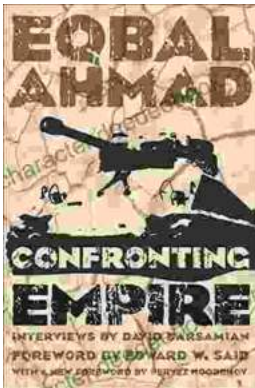
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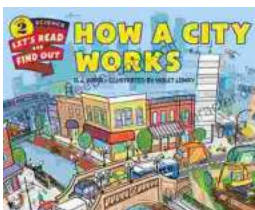
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